14) Find the rate of change $\frac{dd}{dt}$ of the distance between the origin and a moving point on the graph of y = sinx if $dx/dt = 2$ cm/sec.

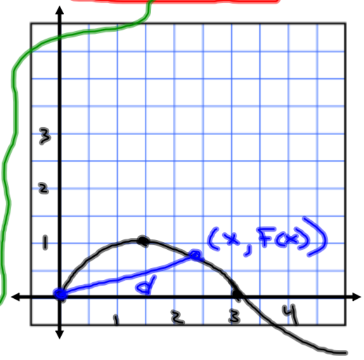
$d = \sqrt{x^2 + f^2(x)}$

$d = \sqrt{x^2 + \sin^2 x}$

$\frac{dd}{dt} = \frac{\cancel{2}x\frac{dx}{dt} + \cancel{2}\sin x\cos x\frac{dx}{dt}}{\cancel{2}\sqrt{x^2+\sin^2 x}}$

$\frac{dd}{dt} = \frac{x + \sin x\cos x}{\sqrt{x^2+\sin^2 x}}\frac{dx}{dt}$

$\boxed{\frac{dd}{dt} = \frac{2x + 2\sin x\cos x}{\sqrt{x^2+\sin^2 x}}}$

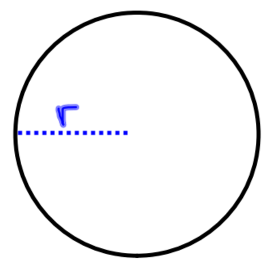


15) The radius r of a circle is increasing at a rate of 3 cm/min. $\frac{dr}{dt}$
Find the rate of change of the area when: $\frac{dA}{dt}$
a) r = 6 cm

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$

$\frac{dA}{dt} = 2\pi(6)(3) = 36\pi\ \frac{cm^2}{min}$



b) r = 24 cm

$\frac{dA}{dt} = 2\pi(24)(3) = 144\pi\ \frac{cm^2}{min}$

16) Let A be the area of a circle of radius r that is changing with respect to time. If dr/dt is constant, is dA/dt constant? Explain.

$\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$

Not constant

still variable

15. ***Area*** The included angle of the two sides of constant equal length s of an isosceles triangle is θ.

$\frac{d\theta}{dt} = \frac{1}{2}$

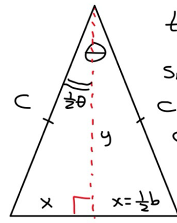
(b) If θ is increasing at the rate of $\frac{1}{2}$ radian per minute, find the rates of change of the area when $\theta = \pi/6$ and $\theta = \pi/3$.

Snapshot

$\frac{dA}{dt}$

$A = (c \cdot \sin\tfrac{1}{2}\theta)(c \cdot \cos\tfrac{1}{2}\theta)$

$A = c^2 \sin\tfrac{1}{2}\theta \cdot \cos\tfrac{1}{2}\theta$



$\tan(\tfrac{1}{2}\theta) = \frac{x}{y}$

$\sin(\tfrac{1}{2}\theta) = \frac{x}{c}$

$\cos(\tfrac{1}{2}\theta) = \frac{y}{c}$

$\frac{dA}{dt} = c^2\cos(\tfrac{1}{2}\theta)\tfrac{1}{2}\frac{d\theta}{dt}\cos(\tfrac{1}{2}\theta) + c^2\sin(\tfrac{1}{2}\theta)\left(-\sin(\tfrac{1}{2}\theta)\tfrac{1}{2}\frac{d\theta}{dt}\right)$ $\quad A = \frac{1}{2}bh = xy$

$\frac{dA}{dt} = c^2\cos^2(\tfrac{1}{2}\theta)\,\tfrac{1}{2}\cdot\tfrac{1}{2} - c^2\sin^2(\tfrac{1}{2}\theta)\,\tfrac{1}{2}\cdot\tfrac{1}{2}$

$\theta = \frac{\pi}{6} \rightarrow 0.217c^2 \ \frac{\text{units}^2}{\text{min}}$

$\theta = \frac{\pi}{3} \rightarrow 0.125c^2 \ \frac{\text{units}^2}{\text{min}}$

$c^2\cos^2(\tfrac{1}{2}\tfrac{\pi}{6})\tfrac{1}{4} - c^2\sin^2(\tfrac{1}{2}\tfrac{\pi}{6})\cdot\tfrac{1}{4}$

$.233c^2 - 0.017c^2$

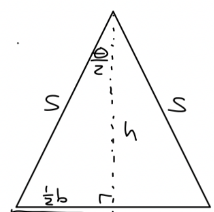
15. ***Area*** The included angle of the two sides of constant equal length s of an isosceles triangle is θ.

(b) If θ is increasing at the rate of $\frac{1}{2}$ radian per minute, find the rates of change of the area when $\theta = \pi/6$ and $\theta = \pi/3$.

Year 24-25

$\sin\frac{\theta}{2} = \frac{\frac{1}{2}b}{s}$ $\qquad \cos\frac{\theta}{2} = \frac{h}{s}$

$s\cdot\sin(\frac{\theta}{2}) = \frac{1}{2}b$ $\qquad s\cos\frac{\theta}{2} = h$



$A = \frac{1}{2}bh$

$A = (s\cdot\sin\frac{\theta}{2})(s\cdot\cos\frac{\theta}{2})$

$A = s^2(\sin\frac{\theta}{2})(\cos\frac{\theta}{2})$

$\frac{dA}{dt} = s^2\left[\cos(\frac{\theta}{2})\frac{1}{2}\frac{d\theta}{dt}\cdot\cos\frac{\theta}{2} - \sin\frac{\theta}{2}\sin(\frac{\theta}{2})\frac{1}{2}\frac{d\theta}{dt}\right]$

$= \frac{1}{2}s^2\left(\cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}\right)\frac{d\theta}{dt}$

$= \frac{1}{2}s^2\left[\cos(2\cdot\frac{\theta}{2})\right]\frac{d\theta}{dt} = \frac{1}{2}s^2(\cos\theta)\frac{d\theta}{dt}$

$\frac{d\theta}{dt}$

a) $\theta = \frac{\pi}{6}$ $\qquad \frac{dA}{dt} = \frac{1}{2}s^2(\cos\frac{\pi}{6})(\frac{1}{2})$

$= \frac{1}{2}s^2\cdot\frac{\sqrt{3}}{2}\cdot\frac{1}{2} = \boxed{\frac{\sqrt{3}}{8}s^2 \ \frac{\text{units}^2}{\text{min}}}$

b) $\theta = \frac{\pi}{3}$

$= \frac{1}{2}s^2(\frac{1}{2})(\frac{1}{2}) = \boxed{\frac{1}{8}s^2 \ \frac{\text{units}^2}{\text{min}}}$

Year 25-26

15. ***Area*** The included angle of the two sides of constant equal length s of an isosceles triangle is θ.

$\frac{d\theta}{dt} = \frac{1}{2}$

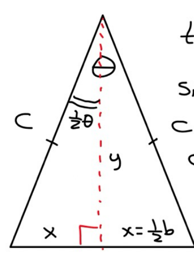
(b) If θ is increasing at the rate of $\frac{1}{2}$ radian per minute, find the rates of change of the area when $\theta = \pi/6$ and $\theta = \pi/3$.

Snapshot

$\frac{dA}{dt}$

$A = (c \cdot \sin\tfrac{1}{2}\theta)(c \cdot \cos\tfrac{1}{2}\theta)$

$A = c^2 \sin\tfrac{1}{2}\theta \cdot \cos\tfrac{1}{2}\theta$



$\tan(\tfrac{1}{2}\theta) = \frac{x}{y}$

$\sin(\tfrac{1}{2}\theta) = \frac{x}{c}$

$\cos(\tfrac{1}{2}\theta) = \frac{y}{c}$

$\frac{dA}{dt} = c^2\cos(\tfrac{1}{2}\theta)\tfrac{1}{2}\frac{d\theta}{dt}\cos(\tfrac{1}{2}\theta) + c^2\sin(\tfrac{1}{2}\theta)\left(-\sin(\tfrac{1}{2}\theta)\tfrac{1}{2}\frac{d\theta}{dt}\right)$ $\quad A = \frac{1}{2}bh = xy$

$\frac{dA}{dt} = c^2\cos^2(\tfrac{1}{2}\theta)\,\tfrac{1}{2}\cdot\tfrac{1}{2} - c^2\sin^2(\tfrac{1}{2}\theta)\,\tfrac{1}{2}\cdot\tfrac{1}{2}$

$\theta = \frac{\pi}{6} \rightarrow 0.217c^2 \ \frac{\text{units}^2}{\text{min}}$

$\theta = \frac{\pi}{3} \rightarrow 0.125c^2 \ \frac{\text{units}^2}{\text{min}}$

$c^2\cos^2(\tfrac{1}{2}\tfrac{\pi}{6})\tfrac{1}{4} - c^2\sin^2(\tfrac{1}{2}\tfrac{\pi}{6})\cdot\tfrac{1}{4}$

$.233c^2 - 0.017c^2$